Seminar 5 Solutions

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Disclaimer

Full solutions are available on my.wbs. All exercises are examinable material, not just the ones we covered in the seminars.

Roadmap

Exercise 1

Exercise 3

Exercise 4

Exercise 6

Exercise 1

Q: The OLS estimator in a time-series setting is unbiased under the first three Gauss-Markov assumptions.

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Q: The OLS estimator in a time-series setting is unbiased under the first three Gauss-Markov assumptions.

1. TS.1: Linearity in parameters
2. TS.2: No perfect collinearity
3. TS.3: Strict exogeneity/ Zero conditional mean

When we add the following two assumptions, the OLS estimator is also BLUE.

4. TS.4: Homoskedasticity
5. TS.5: No serial correlation

Exercise 1

Q: A trending variable cannot be used as a dependent variable in the multiple linear regression model.

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Q: A trending variable cannot be used as a dependent variable in the multiple linear regression model.

- Trending variables *can* be used as dependent variables in a linear regression model.
- However, be cautious when interpreting the results:
  - spurious relationship between y_t and trending explanatory variables.
- Including a time trend in the regression is advisable when dependent and/or independent variables are trending.
- The usual R^2 measure can be misleading when the dependent variable is trending.

Exercise 1

Q: Seasonality is not an issue when using annual time-series observations.

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- Each period represents a year and this is not associated with any season.

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Exercise 3

Weakly Stationary Process: Correlation

Let $\{x_t : t = 1, 2, \ldots, T\}$ be a weakly stationary process.

Define $\gamma_h = \mathrm{Cov}(x_t, x_{t+h})$ for $h \geq 0$. Then $\gamma_0 = \mathrm{Var}(x_t)$. Show that

$$\mathrm{Corr}(x_t, x_{t+h}) = \frac{\gamma_h}{\gamma_0}.$$

Weak (or covariance) Stationarity

A stochastic process $\{x_t : t = 1, 2, \ldots\}$ is said to be weakly stationary if:

$$\mathbb{E}(x_t) = \mu, \quad \mathrm{Var}(x_t) = \sigma^2, \quad \mathrm{Cov}(x_t, x_{t+h}) = f(h).$$

A weakly stationary process is uniquely determined by its mean, variance, and autocovariance function.

Derivation:

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Exercise 4

Suppose that a time-series process $\{x_t : t = 1, 2, \ldots, T\}$ is given by

$$x_t = z + \epsilon_t,$$

for all $t = 1, 2, \ldots, T$, where ϵ_t is an i.i.d. sequence with mean zero and variance σ_ϵ^2. The random variable z is constant over time, and it has mean zero and variance σ_z^2. Furthermore, assume that ϵ_t is uncorrelated with z.

Exercise 4

Q: Find the expected value and variance of x_t. Do your answers depend on t?

Exercise 4

Q: Find $\text{Cov}(x_t, x_{t+h})$ for any t and h. Is x_t a weakly stationary process?

Exercise 4

Q: Show that $\mathrm{Corr}(x_t, x_{t+h}) = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\epsilon^2}$ for any t and h.

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model

$$\text{Return}_t = \beta_0 + \beta_1 \,\text{Return}_{t-1} + u_t, \quad u_t \sim N(0, \sigma^2).$$

Source	SS	df	MS
Model	10.6866231	1	10.6866231
Residual	3059.73817	687	4.45376735
Total	3070.42479	688	4.46282673

Number of obs =	689
F(1, 687) =	2.40
Prob > F =	0.1218
R-squared =	0.0035
Adj R-squared =	0.0020
Root MSE =	2.1104

return	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
return_1	.0588984	.0380231	1.55	0.122	-.0157569	.1335538
_cons	.179634	.0807419	2.22	0.026	.0211034	.3381646

Figure: Predictive Model for Stock Returns

Q: Compute the uncoditional mean and variance of the returns.

Exercise 6

- AR(1) model: $\text{Return}_t = \beta_0 + \beta_1 \, \text{Return}_{t-1} + u_t$.
- uncoditional mean: $\mathbb{E}(\text{Return}_t) = \frac{\beta_0}{1-\beta_1}$.
- uncoditional variance: $\text{Var}(\text{Return}_t) = \frac{\sigma^2}{1-\beta_1^2}$.
- use $\hat{\sigma}^2$ as an estimator of σ^2

$$\hat{\sigma}^2 = \frac{\hat{u}'\hat{u}}{T-k-1}$$