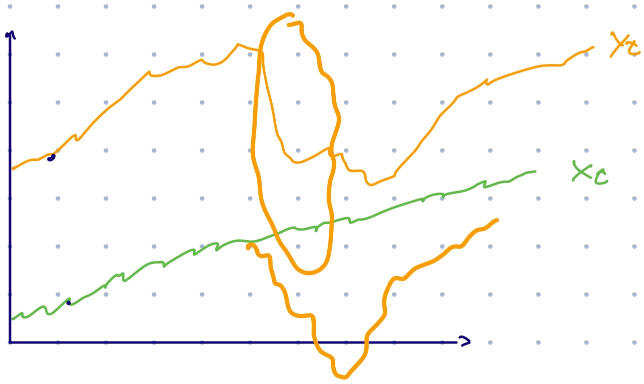
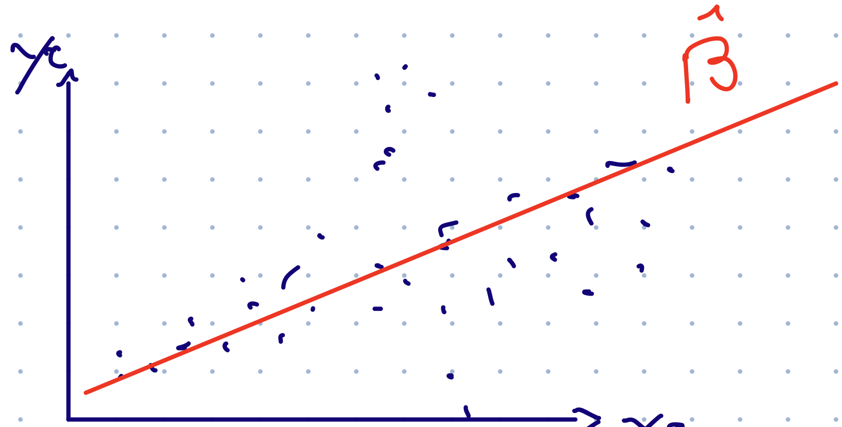



relationship b/w $x_t$ and $y_t$

start $\quad t = 0 \quad X_0 = 0$

$\downarrow$

$t = 1 \quad X_1 = X_0 + e_1$ $\quad$ generate $e_1 \sim (0, 1)$

$X_0 = 0$ $\quad$ $e_1$: 0.045

$x_0 = 0 \quad x_1 = 0 + 0.045 = 0.045$

$t = 2 \quad x_2 = x_1 + e_2$ $\quad$ generate $e_2 \sim (0, 1)$

$x_1 = 0.045$ $\quad$ $e_2$: $-0.20$

$= 0.045 - 0.2$

AR(1) $\quad \hat{\rho} \Rightarrow t\text{ test} \quad \varepsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$

$$x_t = \alpha + \rho\, x_{t-1} + \varepsilon_t$$

$$|\rho| = 1 \longrightarrow x_t = \alpha + x_{t-1} + \varepsilon_t \quad \text{RW}$$

$$\alpha \neq 0 \Rightarrow \text{RW w/drift}$$

$H_0: |\rho| = 1 \Rightarrow$ non stationary

$H_1: |\rho| < 1 \Rightarrow$ stationarity

↳ under $H_0$ $\quad x_t, x_{t-1} \rightarrow$ RW

$$x_t - x_{t-1} = \alpha + (\rho - 1)x_{t-1} + \varepsilon_t$$

$$\underbrace{x_t - x_{t-1}}_{\Delta x_t} = \alpha + \rho x_{t-1} - x_{t-1} + \varepsilon_t$$

$$= \alpha + \underbrace{(\rho - 1)}_{\theta} x_{t-1} + \varepsilon_t$$

$$\Delta x_t = \alpha + \theta x_{t-1} + \varepsilon_t$$

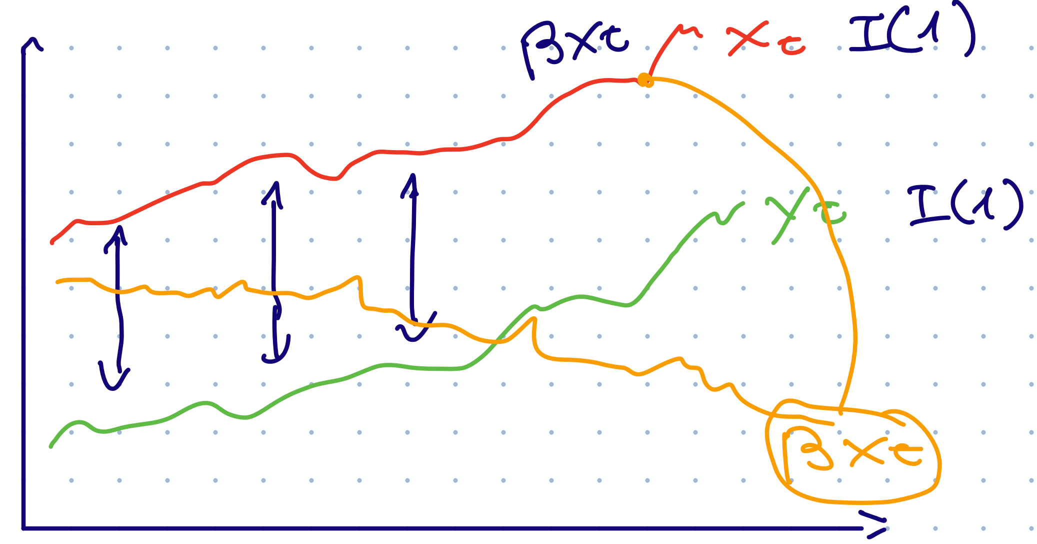
$H_0: \rho = 1 \Rightarrow \theta = 0 \quad \Delta x_t = \alpha + \varepsilon_t$

$H_1: \rho < 1 \Rightarrow \Delta x_t = \alpha + \theta x_{t-1} + \varepsilon_t$ $\quad \theta < 0$

$\hat{\theta} \rightarrow$ compute t-stat

DF distribution

$$t_{stat} < t_{crit.}$$




$$\underbrace{y_t - \boxed{\beta} x_t}_{*} \Rightarrow I(0)$$

↳ stationary → $\beta$ scaling factor such that distance b/w $y_t$ $\beta x_t$ is constant = stationary

$\beta$ exists → $y_t$ $x_t$ cointegrated

need estimate of $\beta$

LS → $y_t = \hat{\alpha} + \hat{\beta} x_t + \hat{u}_t$

$$\hat{u}_t = \underbrace{y_t - \hat{\alpha} - \hat{\beta} x_t}_{*}$$

b

I(0)

DF test $\hat{u}_t$

→ $\Delta \hat{u}_t = \delta_0 + \delta_1 \hat{u}_{t-1} + \dots + \varepsilon_t$