Seminar 7 Solutions

Giulio Rossetti* giuliorossetti94.github.io February 28, 2025

* email: giulio.rossetti.1@wbs.ac.uk

Disclaimer

Full solutions are available on my.wbs. All exercises are examinable material, not just the ones we covered in the seminars.

Exercise 1

Exercise 2

Exercise 3

Exercise 1: Simple Linear Regression (SLR) Model

Consider the savings function:

$$sav = \beta_0 + \beta_1 inc + u, \quad u = \sqrt{inc} \cdot e \quad \mathbf{E}[e] = 0 \quad \mathbf{Var}[e] = \sigma_e^2$$

Assume that e is independent of *inc*.

Show that the zero conditional mean assumption is satisfied.

Derivation

- When we condition on *inc*, \sqrt{inc} becomes a constant.
- Therefore:

$$E[u|inc] = E[\sqrt{inc} \cdot e|inc] = \sqrt{inc}E[e|inc]$$

• Since E[e|inc] = E[e] = 0, it follows that:

$$E[u|inc] = 0.$$

Exercise 1: Violation of Homoskedasticity

Show that the homosked asticity assumption SLR.5 is violated; that is, ${\rm Var}(u|inc)=\sigma_e^2inc.$

• Computing the conditional variance:

$$\operatorname{Var}[u|inc] = \operatorname{Var}[\sqrt{inc} \cdot e|inc]$$

Since variance operators allow constants to be factored out:

$$Var[u|inc] = inc \cdot Var[e|inc]$$
$$= inc \cdot \sigma_e^2.$$

• This shows that variance depends on *inc*, violating the homoskedasticity assumption.

Exercise 1

Exercise 2

Exercise 3

Exercise 2: Multiple Linear Regression (MLR) Model

Model

$$GPA = \beta_0 + \beta_1 study + \beta_2 sleep + \beta_3 work + \beta_4 leisure + u$$

does it make sense to hold *sleep*, *work*, and *leisure* fixed, while changing *study*?

- No. By definition, study + sleep + work + leisure = 168.
- If we change *study*, at least one other category must also change to maintain the total sum.

Violations of Gauss-Markov Assumptions

 study is a perfect linear combination of other regressors. This holds for every observation, violating MLR.3.

$$study = 168 - sleep - work - leisure.$$

How to fix this?

• Drop one of the independent variables, say leisure:

$$GPA = \beta_0 + \beta_1 study + \beta_2 sleep + \beta_3 work + u.$$

- β₁: change in GPA when *study* increases by one hour, holding *sleep*, *work*, and *u* constant.
- If *sleep* and *work* are fixed but $study \uparrow$, then $leisure \downarrow$ by one hour.

Exercise 1

Exercise 2

Exercise 3

Dickey Fuller Test

Exercise 1

Exercise 2

Exercise 3

Exercise 4: Error Correction Model

Suppose that the process $\{(x_t, y_t) : t = 0, 1, 2, ...\}$ satisfies the equations:

$$y_t = \beta x_t + u_t$$

and

$$\Delta x_t = \gamma \Delta x_{t-1} + \nu_t,$$

where:

- $E[u_t|I_{t-1}] = E[\nu_t|I_{t-1}] = 0$,
- I_{t-1} contains information on x and y dated at time t-1 and earlier,
- $\beta \neq 0$, and $|\gamma| < 1$ (so that x_t , and therefore y_t , is I(1)).

Exercise 4: Error Correction Model

Show that the given equations imply an error correction model:

$$\Delta y_t = \gamma_1 \Delta x_{t-1} + \delta(y_{t-1} - \beta x_{t-1}) + e_t,$$

where:

$$\gamma_1 = \beta \gamma, \quad \delta = -1, \quad e_t = \beta \nu_t + u_t.$$

Step 1: Expressing Δy_t

Start by subtracting y_{t-1} from both sides of $y_t = \beta x_t + u_t$:

$$\Delta y_t = \beta x_t - y_{t-1} + u_t.$$

Step 2: Introducing βx_{t-1}

• Add and subtract βx_{t-1} on the right-hand side:

$$\Delta y_t = \beta \Delta x_t - (y_{t-1} - \beta x_{t-1}) + u_t.$$

• This isolates the error correction term $y_{t-1} - \beta x_{t-1}$.

Now substitute $\Delta x_t = \gamma \Delta x_{t-1} + \nu_t$ into the equation:

$$\Delta y_t = \beta (\gamma \Delta x_{t-1} + \nu_t) - (y_{t-1} - \beta x_{t-1}) + u_t.$$

Step 3: Final Expression

• Distribute β in the equation:

$$\Delta y_t = \beta \gamma \Delta x_{t-1} - (y_{t-1} - \beta x_{t-1}) + (\beta \nu_t + u_t).$$

• Recognizing the key components:

$$\gamma_1 = \beta \gamma, \quad \delta = -1, \quad e_t = \beta \nu_t + u_t.$$

• Thus, we obtain the final error correction model:

$$\Delta y_t = \gamma_1 \Delta x_{t-1} + \delta(y_{t-1} - \beta x_{t-1}) + e_t.$$